Spillovers under Common Shocks in Panel Data: Estimation and Inference

Qiyang Yu

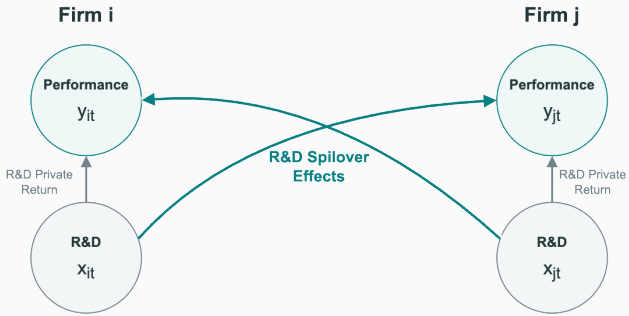
November 10, 2026

Motivation



$$y_{it} = \rho x_{it} + e_{it}$$

Motivation



$$y_{it} = \rho x_{it} + \gamma_{ij} x_{jt} + e_{it}$$

Motivation

Empirical evidence of spillovers in various contexts:

- **Firm R&D Spillovers**: Jaffe, 1986; Bloom et al., 2013; Lucking et al., 2018; Arora et al., 2021; Arque-Castells and Spulber, 2022
- **Peer effects in education**: Sacerdote, 2001; Golsteyn et al., 2021; Barwick et al., 2026
- **Cross-predictability of asset characteristics**: Kelly et al., 2023; Avramov et al., 2026
- **Interest rate spillovers**: Ilzetzki and Jin, 2021; Camara, 2025

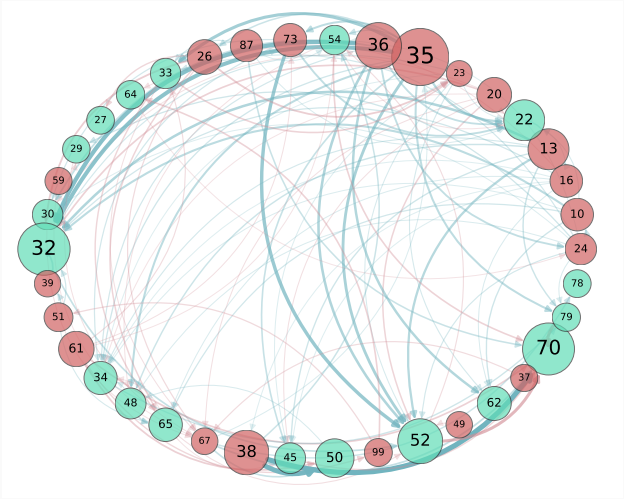
Motivation

Empirical evidence of spillovers in various contexts:

- **Firm R&D Spillovers**: Jaffe, 1986; Bloom et al., 2013; Lucking et al., 2018; Arora et al., 2021; Arque-Castells and Spulber, 2022
- **Peer effects in education**: Sacerdote, 2001; Golsteyn et al., 2021; Barwick et al., 2026
- **Cross-predictability of asset characteristics**: Kelly et al., 2023; Avramov et al., 2026
- **Interest rate spillovers**: Ilzetzki and Jin, 2021; Camara, 2025

Q: Who affects whom? By how much?

Estimated industry spillover network



Selected SIC2 industries

- **22** Textile mill products
- **32** Stone, clay, glass and concrete products
- **35** Machinery and computer equipment
- **36** Electrical and electronic equipment
- **52** Building materials and hardware retail
- **70** Hotels and other lodging

Arrows: R&D sender → receiver.

- Positive effect
- Negative effect

Nodes: total absolute effects.

- Net sender
- Net receiver

Challenge 1: A high-dimensional spillover network

Each unit may receive spillovers from $N-1$ other units: potentially too many coefficients to estimate reliably from T observations.

Common empirical approach

Summarize interactions using pre-specified network weights:

$$y_{it} = \rho x_{it} + \gamma\, Spill_{it} + e_{it}, \qquad Spill_{it} = \sum_{j \neq i} \omega_{ij} x_{jt}.$$

Weights: geography, or group membership, technology/product-market similarity.

Challenge 1: A high-dimensional spillover network

Each unit may receive spillovers from $N-1$ other units: potentially too many coefficients to estimate reliably from T observations.

Common empirical approach

Summarize interactions using pre-specified network weights:

$$y_{it} = \rho x_{it} + \gamma\, Spill_{it} + e_{it}, \qquad Spill_{it} = \sum_{j \neq i} \omega_{ij} x_{jt}.$$

Weights: geography, or group membership, technology/product-market similarity.

Challenge 1: A high-dimensional spillover network

Each unit may receive spillovers from $N-1$ other units: potentially too many coefficients to estimate reliably from T observations.

Common empirical approach

Summarize interactions using pre-specified network weights:

$$y_{it} = \rho x_{it} + \gamma\, Spill_{it} + e_{it}, \qquad Spill_{it} = \sum_{j \neq i} \omega_{ij} x_{jt}.$$

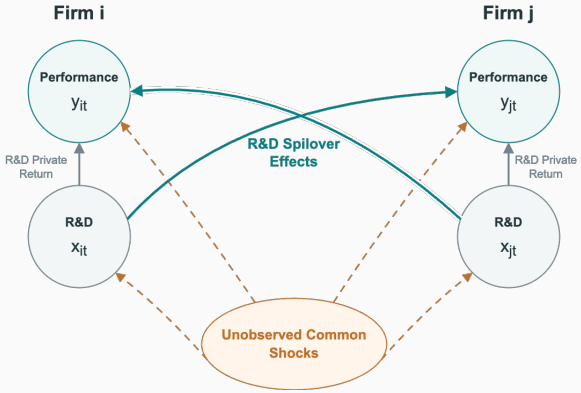
Weights: geography, or group membership, technology/product-market similarity.

Limitations

- Chosen weights may misrepresent the intended spillover channel.
- Omitting correlated channels can bias the coefficients. (Ritzwoller, 2026)

Challenge 2: Unobserved common shocks

Unobserved common shocks can affect both outcomes and covariates. Their influence may be mistaken for private or spillover effects. (Manski, 1993)



Shared shocks generate comovement even without spillovers.

Challenge 2: Unobserved common shocks

Unobserved common shocks can affect both outcomes and covariates. Their influence may be mistaken for private or spillover effects. (Manski, 1993)

Time fixed effects

Absorb common time variation.

Limitation

Impose the same response to that variation across units.

Challenge 2: Unobserved common shocks

Unobserved common shocks can affect both outcomes and covariates. Their influence may be mistaken for private or spillover effects. (Manski, 1993)

Time fixed effects

Absorb common time variation.

Limitation

Impose the same response to that variation across units.

Instrumental variables

Use covariate variation unrelated to unobserved outcome shocks.

Limitation

Valid instruments can be difficult to find with many spillover regressors.

Challenge 2: Unobserved common shocks

Unobserved common shocks can affect both outcomes and covariates. Their influence may be mistaken for private or spillover effects. (Manski, 1993)

Time fixed effects

Absorb common time variation.

Limitation

Impose the same response to that variation across units.

Instrumental variables

Use covariate variation unrelated to unobserved outcome shocks.

Limitation

Valid instruments can be difficult to find with many spillover regressors.

An additional identification challenge

With an unknown network, common variation may be attributed either to spillovers or to common shocks—even when those shocks enter as time fixed effects.

My strategy

Additional challenges:

- Potentially large errors from the network estimation step.
- Pseudo inverse of rank degenerate matrices.

My strategy

Additional challenges:

- Potentially large errors from the network estimation step.
- Pseudo inverse of rank degenerate matrices.

Strategy

Remove common effects

Common shocks affect units with different strengths.

→

Learn the spillover network

Assume only a few peers matter.

→

Learn private effects

Post-selection approach

My Paper

1. Network recovery

Estimate unknown bilateral spillovers while accounting for common shocks.

2. Valid inference

Bilateral and common private effects; joint tests of incoming spillovers.

3. Technical contribution

Leave-one-out strategy for pseudo-inverse rank degeneracy.

4. U.S. industry R&D application

Common shocks matter for estimated private returns and networks.

Literature

Related literature	Main focus	Beyond that scope
Unknown spillover networks Manresa (2015); Okui et al. (2025).	Bilateral spillover effects	Separation from correlated common shocks
Panel models with common factors Pesaran (2006); Bai (2009); Moon and Weidner (2015, 2026)...	Slope coefficients and common components CCE, PCA, nuclear norm	Recovery of an unknown bilateral network
Karabiyik, Reese, and Westerlund (2017)	Pseudo-inverse rank degeneracy in CCE estimators	Pseudo-inverse in PCA-based factor estimators

Model Setup: Bilateral Spillovers

A panel model with bilateral spillovers:

$$y_{it} = \rho x_{it} + \sum_{j \neq i} \gamma_{ij} x_{jt} + \beta' w_{it} + \lambda_i' f_t + \epsilon_{it}.$$

- x_{it}: covariate of interest for unit i at time t;
- y_{it}: outcome of unit i at time t;
- w_{it}: other controls.
- f_t: unobserved common factors that affect both x_{it} and y_{it};
- λ_i: heterogeneous effects of common factors on unit i;

Model Setup: Bilateral Spillovers

A panel model with bilateral spillovers:

$$y_{it} = \rho x_{it} + \sum_{j \neq i} \gamma_{ij} x_{jt} + \beta' w_{it} + \lambda_i' f_t + \epsilon_{it}.$$

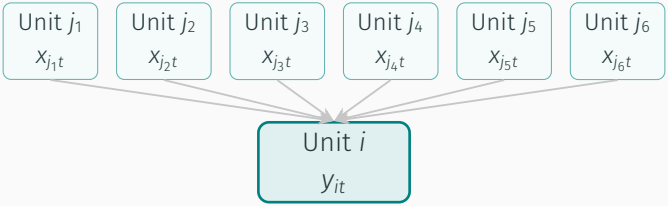
- x_{it}: covariate of interest for unit i at time t;
- y_{it}: outcome of unit i at time t;
- w_{it}: other controls.
- f_t: unobserved common factors that affect both x_{it} and y_{it};
- λ_i: heterogeneous effects of common factors on unit i;

Parameters of interest:

- ρ: common private effect of x_{it};
- γ_{ij}: spillover coefficient from unit j to unit i.

Model setup: Few spillover sources per unit

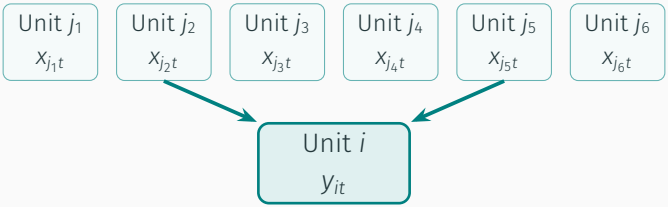
$$y_{it} = \rho x_{it} + \sum_{j \neq i} \gamma_{ij} x_{jt} + \beta' w_{it} + \lambda_i' f_t + \epsilon_{it}.$$



Many potential senders for each receiving unit.

Model setup: Few spillover sources per unit

$$y_{it} = \rho x_{it} + \sum_{j \neq i} \gamma_{ij} x_{jt} + \beta' w_{it} + \lambda_i' f_t + \epsilon_{it}.$$



Each unit receives spillovers mainly from a subset of other units.

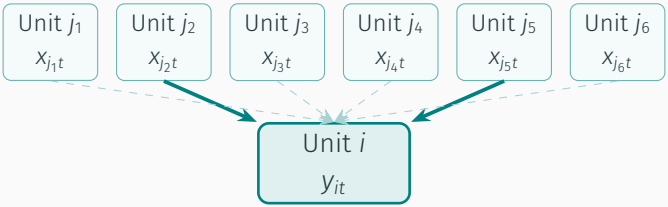
The size of this set can grow slowly with T.

The sources remain unknown.

Sparsity restricts incoming links but allows **star units** with many outgoing links.

Model setup: Few spillover sources per unit

$$y_{it} = \rho x_{it} + \sum_{j \neq i} \gamma_{ij} x_{jt} + \beta' w_{it} + \lambda_i' f_t + \epsilon_{it}.$$



Each unit receives spillovers mainly from a subset of other units.

The size of this set can grow slowly with T.

The sources remain unknown.

Sparsity restricts incoming links but allows **star units** with many outgoing links.

Robustness: Simulations support the algorithm's performance under approximate sparsity.

Identification: Spillovers or common shocks?

$$y_{it} = \rho x_{it} + \sum_{j \neq i} \gamma_{ij} x_{jt} + \beta' w_{it} + \lambda_i' f_t + \epsilon_{it}.$$

The same comovement can reflect bilateral spillovers or responses to common shocks.

A simple example

Suppose unit j's covariate x_{jt} moves only with the common shocks:

$$x_{jt} = a_j' f_t.$$

Identification: Spillovers or common shocks?

$$y_{it} = \rho x_{it} + \sum_{j \neq i} \gamma_{ij} x_{jt} + \beta' w_{it} + \lambda_i' f_t + \epsilon_{it}.$$

The same comovement can reflect bilateral spillovers or responses to common shocks.

A simple example

Suppose unit j's covariate x_{jt} moves only with the common shocks:

$$x_{jt} = a_j' f_t.$$

Its contribution to unit i's outcome can then be written as

$$\underbrace{\gamma_{ij} x_{jt}}_{\text{spillover}} + \underbrace{\lambda_i' f_t}_{\text{common shocks}} = \underbrace{(\lambda_i + \gamma_{ij} a_j)' f_t}_{\text{same variation, different attribution}}.$$

Identification: Spillovers or common shocks?

$$y_{it} = \rho x_{it} + \sum_{j \neq i} \gamma_{ij} x_{jt} + \beta' w_{it} + \lambda_i' f_t + \epsilon_{it}.$$

The same comovement can reflect bilateral spillovers or responses to common shocks.

A simple example

Suppose unit j's covariate x_{jt} moves only with the common shocks:

$$x_{jt} = a_j' f_t.$$

Its contribution to unit i's outcome can then be written as

$$\underbrace{\gamma_{ij} x_{jt}}_{\text{spillover}} + \underbrace{\lambda_i' f_t}_{\text{common shocks}} = \underbrace{(\lambda_i + \gamma_{ij} a_j)' f_t}_{\text{same variation, different attribution}}.$$

Need additional variation to separate the two!

General case Case 2

Identification: Use variation beyond common shocks

Model each covariate as a common component plus unit-specific variation.

$$x_{it} = \underbrace{f_t'\eta_{x,i} + f_{x,t}'b_{x,i}}_{\text{common component}} + \underbrace{u_{x,it}}_{\text{unit-specific variation}},$$

$$w_{it} = \underbrace{f_t'\eta_{w,i} + f_{w,t}'b_{w,i}}_{\text{common component}} + \underbrace{u_{w,it}}_{\text{unit-specific variation}}.$$

Identification: Use variation beyond common shocks

Model each covariate as a common component plus unit-specific variation.

$$x_{it} = \underbrace{f_t'\eta_{x,i} + f_{x,t}'b_{x,i}}_{\text{common component}} + \underbrace{u_{x,it}}_{\text{unit-specific variation}},$$

$$w_{it} = \underbrace{f_t'\eta_{w,i} + f_{w,t}'b_{w,i}}_{\text{common component}} + \underbrace{u_{w,it}}_{\text{unit-specific variation}}.$$

Identify common factor space from X, W → Remove common components → Exogenous variations identify effects

Related regressor factor structures are used in the CCE literature (Pesaran, 2006).

Factor structures are used in empirical studies (of R&D).

(e.g., Eberhardt et al., 2013; Eberhardt and Teal, 2013; Ertur and Musolesi, 2017, among others.)

The model: Private effects, spillovers, and common shocks

Full model

$$y_{it} = \underbrace{\rho x_{it} + \beta' w_{it}}_{\text{own inputs}} + \underbrace{\sum_{j \neq i} \gamma_{ij} x_{jt}}_{\text{bilateral spillovers}} + \underbrace{\lambda_i' f_t}_{\text{common shocks}} + \epsilon_{it},$$

$$x_{it} = \eta_{x,i}' f_t + b_{x,i}' f_{x,t} + u_{x,it},$$

$$w_{it} = \eta_{w,i}' f_t + b_{w,i}' f_{w,t} + u_{w,it}.$$

- γ_{ij} is unit-wise sparse.

Inference targets

γ_{ij}: bilateral spillovers ρ: common private effect β: control coefficients.

Estimation: Basic Idea

Following the identification strategy, estimation proceeds in three steps.



Step 1: Estimate the factor space

Write $T \times N$ matrix form:

$$X = F\eta_X' + F_X B_X' + U_X := G_X C_X' + U_X, \qquad W = F\eta_W' + F_W B_W' + U_W := G_W C_W' + U_W$$

The space spanned by $\hat{G}$ can be estimated by principal component analysis (PCA):

1. $\hat{G}_X \Leftarrow PCA(X)$; $\hat{G}_W \Leftarrow PCA(W)$;
2. $\hat{G} = \left(\hat{G}_X, \hat{G}_W\right)$.

Step 1: Estimate the factor space

Write $T \times N$ matrix form:

$$X = F\eta_X' + F_X B_X' + U_X := G_X C_X' + U_X, \qquad W = F\eta_W' + F_W B_W' + U_W := G_W C_W' + U_W$$

The space spanned by $\hat{G}$ can be estimated by principal component analysis (PCA):

1. $\hat{G}_X \Leftarrow PCA(X)$; $\hat{G}_W \Leftarrow PCA(W)$;
2. $\hat{G} = \left(\hat{G}_X, \hat{G}_W\right)$.

Intuition:

$$\hat{\Sigma}_{XX'} := \frac{1}{NT} XX' = \underbrace{\frac{1}{T} G_X \left(\frac{1}{N} B_X' B_X\right) G_X'}_{\text{Signal term}} + \underbrace{\frac{1}{NT} U_X U_X' + \frac{1}{NT} G_X B_X' U_X' + \frac{1}{NT} U_X B_X G_X'}_{\text{Small-order terms}}.$$

Under mild conditions, the space spanned by G_X can be well estimated by the top eigenvectors of $\hat{\Sigma}_{XX'}$.

Step 1 - Why not CCE?

Q: Why PCA is preferred over CCE estimator?

A: CCE is first-moment-based, while PCA is second-moment-based.

Step 1 - Why not CCE?

Q: Why PCA is preferred over CCE estimator?

A: CCE is first-moment-based, while PCA is second-moment-based.

- Let $Z_i = (X_i, W_i)$ and write

$$Z_i = G\Pi_i' + U_i, \quad i = 1, \ldots, N.$$

Then CCE estimator proxies factor by

$$\bar{Z} := \frac{1}{N}\sum_{i=1}^{N} Z_i = G\left(\frac{1}{N}\sum_{i=1}^{N}\Pi_i'\right) + \underbrace{\frac{1}{N}\sum_{i=1}^{N} U_i}_{\text{Small order}}.$$

- CCE estimator proxies factors badly when $\left(\frac{1}{N}\sum_{i=1}^{N}\Pi_i'\right)$ is degenerate, e.g. cross-sectional mean of loadings are close.
- PCA can still recover the factor space in this case as long as the second-moment of loadings is non-degenerate.

Step 2 - Estimate Spillover Parameters

After obtaining $\hat{G}$, I estimate γ_{ij} using LASSO: let $\mathbb{Z}_i = (X_i, W_i) \in \mathbb{R}^{T\times 2}$.

$$\left\{\hat{\gamma}_{ij}\right\}_{j\neq i} = \underset{\gamma_{ij}}{\text{argmin}} \frac{1}{2T} \left\| M_{\hat{\mathbb{Z}},i} M_{\hat{G}} Y_i - \sum_{j\neq i} M_{\hat{\mathbb{Z}},i} M_{\hat{G}} X_j \cdot \gamma_{ij} \right\|_2^2 + \upsilon_i \sum_{j\neq i} |\gamma_{ij}|, \quad \text{for } i = 1, \ldots, N.$$

where $M_{\hat{G}} = I_T - \hat{G}\left(\hat{G}'\hat{G}\right)^+ \hat{G}'$ and $M_{\hat{\mathbb{Z}},i} = I_T - M_{\hat{G}}\mathbb{Z}_i \left(\mathbb{Z}_i' M_{\hat{G}} \mathbb{Z}_i\right)^{-1} \mathbb{Z}_i' M_{\hat{G}}$ and υ_i is a tuning parameter.

Step 2 - Estimate Spillover Parameters

After obtaining $\hat{G}$, I estimate γ_{ij} using LASSO: let $\mathbb{Z}_i = (X_i, W_i) \in \mathbb{R}^{T\times 2}$.

$$\left\{\hat{\gamma}_{ij}\right\}_{j\neq i} = \underset{\gamma_{ij}}{\text{argmin}} \frac{1}{2T}\left\| M_{\hat{\mathbb{Z}},i} M_{\hat{G}} Y_i - \sum_{j\neq i} M_{\hat{\mathbb{Z}},i} M_{\hat{G}} X_j \cdot \gamma_{ij} \right\|_2^2 + \upsilon_i \sum_{j\neq i} |\gamma_{ij}|, \quad \text{for } i = 1, \ldots, N.$$

where $M_{\hat{G}} = I_T - \hat{G}\left(\hat{G}'\hat{G}\right)^{+}\hat{G}'$ and $M_{\hat{\mathbb{Z}},i} = I_T - M_{\hat{G}}\mathbb{Z}_i\left(\mathbb{Z}_i' M_{\hat{G}} \mathbb{Z}_i\right)^{-1}\mathbb{Z}_i' M_{\hat{G}}$ and υ_i is a tuning parameter.

For the propose of inference, I propose the following debiased LASSO estimator: let $\gamma_{i:} = (\gamma_{i1}, \ldots, \gamma_{ii-1}, \gamma_{ii+1}, \ldots, \gamma_{iN})'$,

$$\hat{\gamma}_{i:}^{(db)} = \hat{\gamma}_{i:} + \frac{1}{T}\hat{\Theta}_{-i} X_{-i}' M_{\hat{G}} M_{\hat{\mathbb{Z}},i} M_{\hat{G}} \left(Y_i - X_{-i}\hat{\gamma}_{i:}\right),$$

where $\hat{\Theta} = \text{diag}\left(\frac{1}{T}X' M_{\hat{G}} X\right)^{-1}$ and $\hat{\Theta}_{-i}$ is $\hat{\Theta}$ excluding the i-th diagonal element.

Step 3 - Estimate Private Effects

I propose the following post-selection estimator for the private effects: Given $\hat{\gamma}_{ij}^{(db)}$ and $M_{\hat{G}}$,

1. For each i,
   - For each i, run adaptive LASSO to select the support set $\hat{S}_i$, using $\hat{\gamma}_{ij}^{(db)}$ as the weights for adaptive LASSO:

$$\left\{\hat{\gamma}_{ij}^{(al)}\right\}_{j\neq i} = \underset{\gamma_{ij}}{\text{argmin}} \frac{1}{2T}\left\| M_{\mathbb{Z},i}M_{\hat{G}}Y_i - \sum_{j\neq i} M_{\mathbb{Z},i}M_{\hat{G}}X_j \cdot \gamma_{ij}\right\|_2^2 + \upsilon_i^{(al)} \sum_{j\neq i} \frac{|\gamma_{ij}|}{|\hat{\gamma}_{ij}^{(db)}|}.$$

   - Run OLS on the selected supports to get $\hat{\gamma}_{ij}^{(ps)}$.
2. Let $\theta = [\rho\ \beta]'$. Estimate θ by

$$\hat{\theta} = \left(\sum_{i=1}^{N} \mathbb{Z}_i' M_{\hat{G}} \mathbb{Z}_i\right)^{-1} \sum_{i=1}^{N} \mathbb{Z}_i' M_{\hat{G}} \left(Y_i - \sum_{j\in\hat{S}_i} X_j \cdot \hat{\gamma}_{ij}^{(ps)}\right).$$

γ factor

Theory: Assumptions

Key assumptions for the theoretical analysis:

- Errors in (Y, X, W) are cross-sectionally i.i.d..
- $(U_X, U_W) \perp G$ and $\epsilon \perp (G, U_X, U_W)$.
- Factors are strong and the numbers of factors are fixed.
- Each unit receives spillovers from a slowly growing number of units.
- Rate conditions: $\frac{\sqrt{N}}{T} \to 0$ and $\frac{\sqrt{T}}{N} \to 0$, ignoring the logarithmic factor.

Additional distributional and time-series conditions apply.

Details

Pseudo-inverse rank-degeneracy problem

Recall that $\hat{G} = \left(\hat{G}_X, \hat{G}_W\right)$.

- G_X and G_W could share common factors, which leads to rank degeneracy of $\hat{G}$.
- The projection matrix $M_{\hat{G}}$ contains the pseudo-inverse $\left(\frac{1}{T}\hat{G}'\hat{G}\right)^{+}$.

Pseudo-inverse rank-degeneracy problem

Recall that $\hat{G} = \left(\hat{G}_X, \hat{G}_W\right)$.

- G_X and G_W could share common factors, which leads to rank degeneracy of $\hat{G}$.
- The projection matrix $M_{\hat{G}}$ contains the pseudo-inverse $\left(\frac{1}{T}\hat{G}'\hat{G}\right)^+$.

Problem: pseudo-inverse is NOT continuous at the point of rank jump.

- Decomposing $P_{\hat{G}}$ into several terms leads to a diverging bound since

$$\left\| \left(\frac{1}{T}\hat{G}'\hat{G}\right)^+ \right\| = \mathcal{O}_p\left(N + T^2\right).$$

Pseudo-inverse rank-degeneracy problem

Recall that $\hat{G} = \left(\hat{G}_X, \hat{G}_W\right)$.

- G_X and G_W could share common factors, which leads to rank degeneracy of $\hat{G}$.
- The projection matrix $M_{\hat{G}}$ contains the pseudo-inverse $\left(\frac{1}{T}\hat{G}'\hat{G}\right)^{+}$.

Problem: pseudo-inverse is NOT continuous at the point of rank jump.

- Decomposing $P_{\hat{G}}$ into several terms leads to a diverging bound since

$$\left\| \left(\frac{1}{T}\hat{G}'\hat{G}\right)^{+} \right\| = \mathcal{O}_p\left(N + T^2\right).$$

Karabiyik et al. (2017): amplify the degenerate component in $\hat{G}$ from CCE estimators by multiplying a diverging sequence.

- This method is too conservative for PCA estimator.
- Does not work if multiple leading terms exist with different rates.

Leave-one-out Strategy

I adopt the idea of leave-one-out analysis in Ding and Chen (2020), Abbe et al. (2022) and Chen et al. (2021).

The core idea is conditional independence. But the implementation depends on the specific term we want to bound. For exmaple:

$$\frac{1}{\sqrt{T}} U'_{x,i} P_{\hat{G}} \epsilon_i = \frac{1}{\sqrt{T}} \underbrace{U'_{x,i} P_{\hat{G}^{(-i)}} \epsilon_i}_{leave-one-out} + \frac{1}{\sqrt{T}} U'_{x,i} \left(P_{\hat{G}} - P_{\hat{G}^{(-i)}} \right) \epsilon_i.$$

- The first term can be bounded by concentration inequalities conditional on $\hat{G}^{(-i)}$, i.e., data with the i-th observation removed.
- The second term can be bounded by a perturbation analysis of singular subspaces.

Theorem 1 (Asymptotic Normality of Debiased LASSO Spillover Estimator)
Under assumptions, suppose the tunning parameter $v_i \geq c\left(\sqrt{\frac{\log N}{T}} + \frac{\log N}{N}\right)$ for some constant $c > 0$, the debiased LASSO spillover estimator is asymptotically normal:

- For any $d \in \mathbb{R}^{N-1}$ with $\|d\|_1 < \infty$

$$\sqrt{T} d' \left(\hat{\gamma}_{i:}^{(db)} - \gamma_{i:}\right) / \sqrt{\hat{\sigma}_{\epsilon,i}^2 d' \hat{\Theta}_{-i} d} \xrightarrow{d} N(0,1);$$

- let $\hat{\mathcal{I}} = \left\| \left(\hat{\Theta}_{-i} \hat{\sigma}_{\epsilon,i}^2\right)^{1/2} \xi \right\|_\infty$, where $\xi \sim N(0, I_{N-1})$,

$$\sup_{c \geq 0} \left| \mathbb{P}\left(\sqrt{T} \left\| \hat{\gamma}_{i:}^{(db)} - \gamma_{i:} \right\|_\infty \leq c\right) - \mathbb{P}\left(\hat{\mathcal{I}} \leq c\right) \right| \to 0,$$

with sample variance $\hat{\sigma}_{\epsilon,i}^2$.

Inference: Does a unit receive any spillovers?

For a given unit i, test all incoming links jointly:

$$H_0 : \gamma_{i:} = 0 \quad \text{vs.} \quad H_1 : \|\gamma_{i:}\|_\infty > 0.$$

Theorem 1 approximates the null distribution of S_i by a Gaussian maximum.

Gaussian bootstrap (hold $\hat{\sigma}_{\epsilon,i}$ and $\hat{\Theta}_{-i}$ fixed)

1. Draw

$$\xi^{(b)} \overset{\text{i.i.d.}}{\sim} N(0, I_{N-1})$$
$$b = 1, \ldots, B$$

→

2. Compute max

$$\hat{\mathcal{I}}_{b,i} = \left\| \hat{\sigma}_{\epsilon,i} \hat{\Theta}_{-i}^{1/2} \xi^{(b)} \right\|_\infty$$

→

3. Obtain critical value

$\hat{c}_{1-\alpha,i}$: empirical $(1-\alpha)$ quantile of $\{\hat{\mathcal{I}}_{b,i}\}_{b=1}^{B}$

Decision: Reject H_0 if $\sqrt{T}\,\|\hat{\gamma}_{i:}^{(db)}\|_\infty > \hat{c}_{1-\alpha,i}$.

Rejection provides evidence that unit i receives at least one spillover.

Theory: Model Selection Consistency

Prop. 1 (Model Selection Consistency) Under the same assumptions, suppose the tuning parameter for adaptive LASSO $\upsilon_i^{(al)}$ satisfies

$$\upsilon_i^{(al)} \to 0, \quad \text{and} \quad T\upsilon_i^{(al)} \geq T^c \quad \text{for some } c > 0,$$

then we have the model selection consistency for the adaptive LASSO:

$$\mathbb{P}\left(\hat{\gamma}_i^{(al)} =_s \gamma_i, \quad \text{for all } i\right) \to 1.$$

It allows us to derive the asymptotic normality for the private effect estimator conditional on the event of correct model selection.

Theorem 2 (Asymptotic Normality of Private Effects Estimator) Under the assumptions of Prop. 1,

$$\sqrt{NT}\left(\hat{\theta} - \theta\right) \xrightarrow{d} N\left(0, \Sigma_{\mathbb{U}\epsilon}\right),$$

where

$$\Sigma_{\mathbb{U}\epsilon} = E\left[\frac{1}{NT}\sum_i \mathbb{U}_i'\mathbb{U}_i\right]^{-1} E\left[\frac{1}{NT}\sum_i \mathbb{U}_i'\mathbb{U}_i\sigma_{\epsilon,i}^2\right] E\left[\frac{1}{NT}\sum_i \mathbb{U}_i'\mathbb{U}_i\right]^{-1}.$$

And the sample variance estimator is consistent under the same conditions:

$$\hat{\Sigma}_{\mathbb{U}\epsilon} - \Sigma_{\mathbb{U}\epsilon} = o_p(1),$$

which enables valid inference for the private effect.

Simulation

DGP:

$$
\begin{aligned}
Y &= X\Gamma' + X \cdot \gamma + W \cdot \beta + \mathbf{1}_T a' + d\mathbf{1}_N' + f\lambda' + \epsilon \\
X &= \mathbf{1}_T a_x' + d\eta_{d,x}' + f\eta_{f,x}' + f_x b_x' + U_x, \\
W &= \mathbf{1}_T a_w' + d\eta_{d,w}' + f\eta_{f,w}' + f_w b_w' + U_w.
\end{aligned}
$$

- $\gamma = 1$, $\beta = 1$.
- Units receive spillovers at magnitude 0.5 from their two nearest neighbors, while the first unit receives no spillovers.
- Factors are correlated with each other.
- Errors are cross-sectionally independent with heteroskedasticity.
- U_x and U_w are correlated.

▸ Details

Simulation Results

	(N, T)	(50,100)	(50,200)	(50,300)	(200,100)	(200,200)	(200,300)
Coverage rates of spillover parameters							
$\gamma_{12} (= 0)$	Debiased LASSO	0.94	0.94	0.95	0.94	0.94	0.95
$\gamma_{21} (= 0.5)$	Debiased LASSO	0.91	0.93	0.93	0.95	0.95	0.95
Rejection rates of the row-wise tests							
$H_0 : \|\gamma_{1:}\|_\infty = 0$	Debiased LASSO	0.07	0.06	0.07	0.07	0.06	0.06
$H_0 : \|\gamma_{2:}\|_\infty = 0$	Debiased LASSO	0.93	1.00	1.00	0.77	1.00	1.00
Coverage rates of private effects							
γ	Non-PS	0.83	0.84	0.84	0.87	0.90	0.91
γ	PS-ALasso	0.90	0.93	0.95	0.90	0.93	0.94
β	Non-PS	0.92	0.94	0.93	0.94	0.94	0.94
β	PS-ALasso	0.92	0.94	0.95	0.93	0.94	0.94

Table 1: 2,000 repetitions per (N, T) design. Nominal coverage: 95%. Test level: 5%. Row-wise rejection rates measure size for unit 1 and power for unit 2. Non-PS omits post-selection; PS-ALasso uses the post-selection refinement.

Network results

Application: R&D private returns and spillovers on productivity

In this section, I study the private returns and spillover effects of R&D on industry aggregate productivity.

- Eberhardt et al. (2013) finds that R&D private returns are overestimated if spillover effects and unobserved common factors are not controlled for.
- I extend their analysis by distinguishing spillover channels from the unobserved common factors.

Application: R&D private returns and spillovers on productivity

In this section, I study the private returns and spillover effects of R&D on industry aggregate productivity.

- Eberhardt et al. (2013) finds that R&D private returns are overestimated if spillover effects and unobserved common factors are not controlled for.
- I extend their analysis by distinguishing spillover channels from the unobserved common factors.

Data: 1991Q1 - 2017Q4, 41 industries (SIC 2-digit), from Compustat Fundamentals Quarterly database,

- $(N, T) = (41, 108)$.

Application: R&D Private and Spillover Returns on Productivity

I estimate the following log-linear productivity model:

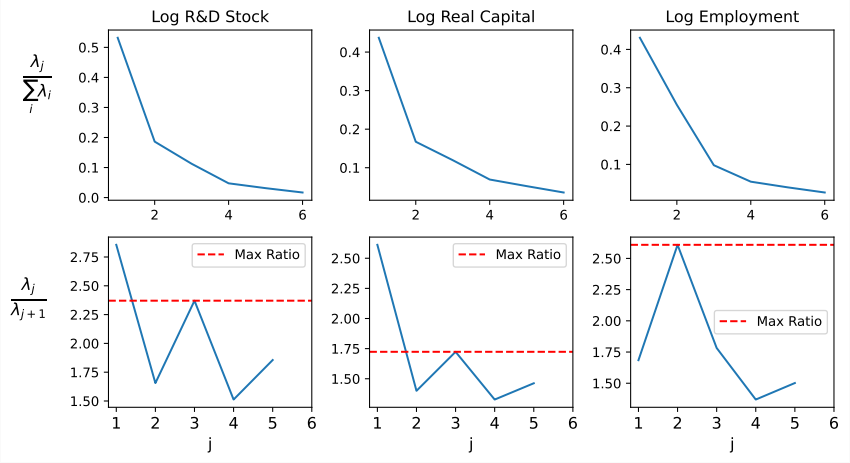
$$y_{it} = c_{it} \cdot \beta_1 + l_{it} \cdot \beta_2 + k_{it} \cdot \rho + \sum_{j \neq i} k_{jt} \cdot \gamma_{ij} + \alpha_i + f_t' \lambda_i + \varepsilon_{it},$$

$$z_{it} = \alpha_{z,i} + f_t' \eta_{z,i} + f_{z,t}' b_{z,i} + u_{z,it}, \quad z \in \{c, l, k\}.$$

where

- y_{it}: log real sales of industry i at time t.
- c_{it}: log capital; l_{it}: log labor; k_{it}: log R&D capital stock.

Application: Eigenvalue Plots



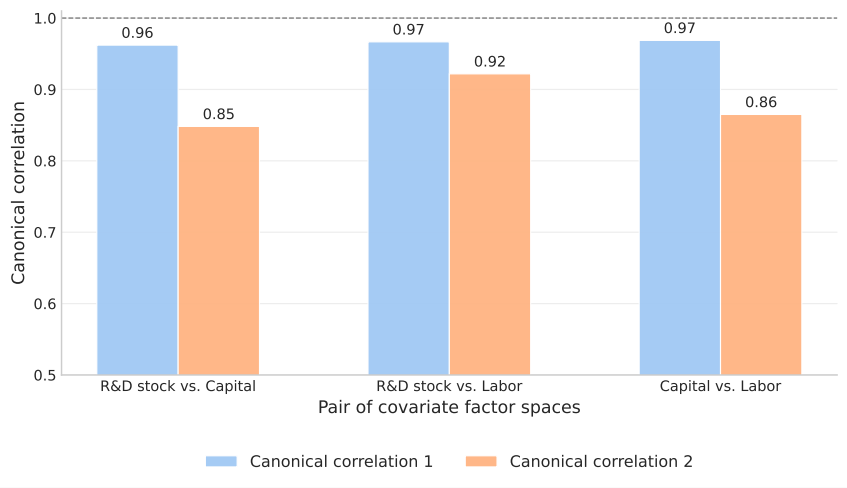
The numbers of factors for R&D, capital, and labor are 3, 3, 2, respectively.

Application: Variance explained by factors

Table 2: Variance explained by estimated factors

	Number of factors	Explained Variance	Residual Variance
Panel A: Covariates			
Log R&D stock	3	83.08%	16.92%
Log real capital	3	72.41%	27.59%
Log employment	2	68.58%	31.42%
Panel B: Outcome			
Log real sales	8	72.60%	27.40%

Application: Factor CCA Plots



Application: Private Return Estimates

Table 3: Coefficient estimates of common parameters in production function

	IFE	Industry FE	Industry FE
R&D Stock	0.014	0.034	0.034
	(0.013)	(0.006)	(0.007)
Capital	0.387	0.449	0.477
	(0.021)	(0.017)	(0.018)
Labor	0.28	0.5	0.549
	(0.029)	(0.024)	(0.027)
Time Cross-fitting	-	No	Yes

Application: R&D private effects and spillovers on productivity

Estimates and inference for spillover parameters:

- Obtain p-values for each pair (i, j) to test $H_0 : \gamma_{ij} = 0$ using the proposed debiased LASSO method.
- Apply the Benjamini-Yekutieli (2001) procedure to control the false discovery rate (FDR) at level 0.05.
- There are 118 significant spillover relationships out of $1,640$ possible pairs.

Application: R&D private effects and spillovers on productivity

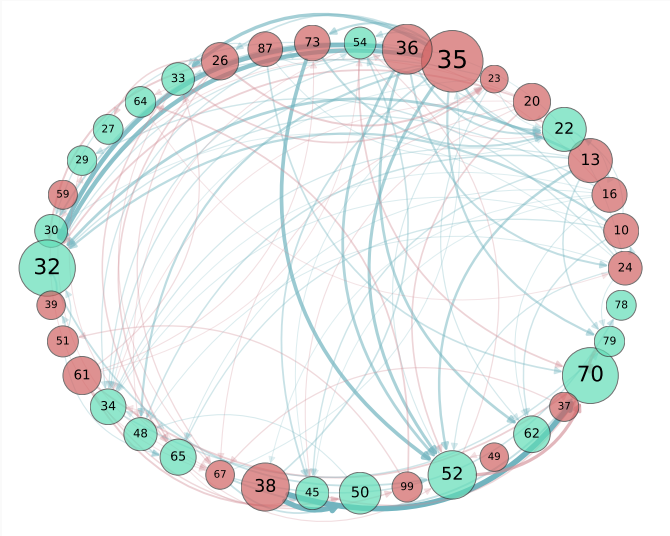
Estimates and inference for spillover parameters:

- Obtain p-values for each pair (i, j) to test $H_0 : \gamma_{ij} = 0$ using the proposed debiased LASSO method.
- Apply the Benjamini-Yekutieli (2001) procedure to control the false discovery rate (FDR) at level 0.05.
- There are 118 significant spillover relationships out of $1,640$ possible pairs.

		Sender		
		Yes	No	Total
Receiver	Yes	25	5	30
	No	8	3	11
	Total	33	8	41

Table 4: Number of significant spillover relationships after controlling FDR at 5% level

Application: R&D Spillover Network



Selected SIC2 industries

- 22 Textile mill products
- 32 Stone, clay, glass and concrete products
- 35 Machinery and computer equipment
- 36 Electrical and electronic equipment
- 38 Instruments and medical, optical and photographic goods
- 52 Building materials and hardware retail
- 70 Hotels and other lodging

Conclusion

- I develop inferential methods for spillover networks and private effects in panel data with unobserved common factors.
- I demonstrate the empirical relevance of the proposed methods in real-world applications.

Conclusion

- I develop inferential methods for spillover networks and private effects in panel data with unobserved common factors.
- I demonstrate the empirical relevance of the proposed methods in real-world applications.

Future Directions:

- Outcome-specific unobserved common factors/shocks.
- Algorithms robust to cross-sectional dependence in the idiosyncratic errors.
- Allowing time variation in spillover effects.
- Imperfect knowledge of the spillover network.

Identification

Consider $Y = X\Gamma' + F\Lambda' + \epsilon$. Suppose $X_{:j} = Fa$ for some $a \in \mathbb{R}^r$, and let e_j be the j-th standard basis vector.

Because $Xe_j = X_{:j} = Fa$, for any $c \in \mathbb{R}^N$ with $c_j = 0$:

Identification

Consider $Y = X\Gamma' + F\Lambda' + \epsilon$. Suppose $X_{:j} = Fa$ for some $a \in \mathbb{R}^r$, and let e_j be the j-th standard basis vector.

Because $Xe_j = X_{:j} = Fa$, for any $c \in \mathbb{R}^N$ with $c_j = 0$:

$$\begin{aligned} X\Gamma' + F\Lambda' &= X\left(\Gamma' - e_j c' + e_j c'\right) + F\Lambda' \\ &= X\left(\Gamma' - e_j c'\right) + F\left(\Lambda' + ac'\right) \\ &:= X\widetilde{\Gamma}' + F\widetilde{\Lambda}'. \end{aligned}$$

- $c_j = 0$ preserves the diagonal.
- $\widetilde{\Gamma} - \Gamma = -ce_j'$ changes at most one coefficient per row. Thus, both matrices can remain row-sparse if we consider row-sparse Γ.
- The same argument applies to sparse sets of X's columns.

Identification

Consider $Y = X\Gamma' + F\Lambda' + \epsilon$. Suppose $X_{:j} = Fa$ for some $a \in \mathbb{R}^r$, and let e_j be the j-th standard basis vector.

Because $Xe_j = X_{:j} = Fa$, for any $c \in \mathbb{R}^N$ with $c_j = 0$:

$$\begin{aligned} X\Gamma' + F\Lambda' &= X\left(\Gamma' - e_j c' + e_j c'\right) + F\Lambda' \\ &= X\left(\Gamma' - e_j c'\right) + F\left(\Lambda' + ac'\right) \\ &:= X\widetilde{\Gamma}' + F\widetilde{\Lambda}'. \end{aligned}$$

- $c_j = 0$ preserves the diagonal.
- $\widetilde{\Gamma} - \Gamma = -ce_j'$ changes at most one coefficient per row. Thus, both matrices can remain row-sparse if we consider row-sparse Γ.
- The same argument applies to sparse sets of X's columns.

Factor structure reveals the common space; idiosyncratic variation identifies spillovers.

Equivalent Representations

Consider $Y = X\Gamma' + F\Lambda' + \epsilon$. Suppose $X_{:S} b_S = Fa$ for some $a \in \mathbb{R}^r$ and $b_S \in \mathbb{R}^{|S|}$ for a subset S of columns, and let b expand b_S to $\mathbb{R}^N$ with zeros outside of S.

Because $Xb = Fa$, for any $c \in \mathbb{R}^N$ with $c_S = 0$:

Equivalent Representations

Consider $Y = X\Gamma' + F\Lambda' + \epsilon$. Suppose $X_{:S} b_S = Fa$ for some $a \in \mathbb{R}^r$ and $b_S \in \mathbb{R}^{|S|}$ for a subset S of columns, and let b expand b_S to $\mathbb{R}^N$ with zeros outside of S.

Because $Xb = Fa$, for any $c \in \mathbb{R}^N$ with $c_S = 0$:

$$\begin{aligned} X\Gamma' + F\Lambda' &= X(\Gamma' - bc' + bc') + F\Lambda' \\ &= X(\Gamma' - bc') + F(\Lambda' + ac') \\ &:= X\widetilde{\Gamma}' + F\widetilde{\Lambda}'. \end{aligned}$$

- $c_S = 0$ preserves the diagonal.
- The value of c can be chosen to preserve row-sparsity or row-approximate-sparsity of Γ.
- This subset exists w.p. 1 when $N > T - r$.

◂ Back

Another Equivalent Representation

Fix unit j, suppose there exists $d \in \mathbb{R}^r$ such that $\lambda_j' d = 0$. Then for all unit i,

$$\begin{aligned}\gamma_{ij} x_{jt} + \lambda_i' f_t &= \gamma_{ij} x_{jt} + \lambda_i' d x_{jt} - \lambda_i' d x_{jt} + \lambda_i' f_t \\ &= (\gamma_{ij} + \lambda_i' d) x_{jt} + \lambda_i' (f_t - d x_{jt}) \\ &= \tilde{\gamma}_{ij} x_{jt} + \lambda_i' \tilde{f}_t.\end{aligned}$$

Another Equivalent Representation

Fix unit j, suppose there exists $d \in \mathbb{R}^r$ such that $\lambda_j' d = 0$. Then for all unit i,

$$\begin{aligned}\gamma_{ij} x_{jt} + \lambda_i' f_t &= \gamma_{ij} x_{jt} + \lambda_i' d x_{jt} - \lambda_i' d x_{jt} + \lambda_i' f_t \\ &= (\gamma_{ij} + \lambda_i' d) x_{jt} + \lambda_i' (f_t - d x_{jt}) \\ &= \tilde{\gamma}_{ij} x_{jt} + \lambda_i' \tilde{f}_t.\end{aligned}$$

- Private effect stays the same for all units since $\rho_j = \rho + \lambda_j' d = \rho$.
- Sparsity is maintained since only one effect per row is changed.
- Such d exists when $r \geq 2$.
- Identification failure occurs when f_t is not identifiable.

◂ Back

IV with Strong Factor Structure

A simple homogeneous panel model with interactive fixed effects:

$$
\begin{aligned}
y_{it} &= x_{it}\beta + f_t\lambda_i + \epsilon_{it}, \\
x_{it} &= f_t\eta_i + g_t b_i + u_{it}, \\
z_{it} &= g_t c_i + v_{it},
\end{aligned}
$$

Assume that $\mathbb{E}\left[\epsilon_{it}|c_i, g_t, v_{it}\right] = \mathbb{E}\left[f_t|c_i, g_t, v_{it}\right] = 0$ and hence z_{it} is a valid instrument for the endogeneous variable x_{it}. The 2SLS estimator of β satisfies

$$
\hat{\beta}^{(2SLS)} - \beta = \frac{\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T} z_{it}\left(f_t\lambda_i + \epsilon_{it}\right)}{\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T} z_{it}x_{it}},
$$

The first term on the numerator:

$$\begin{aligned}\frac{1}{NT}\sum_i\sum_t z_{it}f_t\lambda_i &= \frac{1}{NT}\sum_i\sum_t (g_t c_i + v_{it})f_t\lambda_i = \\ &= \left(\frac{1}{N}\sum_i c_i\lambda_i\right)\left(\frac{1}{T}\sum_t g_t f_t\right) + \frac{1}{NT}\sum_i\sum_t v_{it}f_t\lambda_i, \\ &= \left(\frac{1}{N}\sum_i c_i\lambda_i\right) O_p\left(T^{-1/2}\right) + O_p\left((NT)^{-1/2}\right).\end{aligned}$$

If c_i and λ_i have non-zero means or are highly-correlated, then $\frac{1}{N}\sum_i c_i\lambda_i = O_p(1)$ and hence the convergence rate of the 2SLS estimator is $O_p\left(T^{-1/2}\right)$.

Back

Theory: Assumptions

Some key assumptions for the theoretical analysis:

1. Cross-sectional independence:
   - $\{\epsilon_{it}, U_{X,it}, U_{W,it}\}$ are i.i.d. across i.
2. Exogeneity:
   - $(U_x, U_w) \perp G$ and $\epsilon \perp (G, U_x, U_w)$.
3. Sparse Spillover Effects and Fixed Number of Factors:
   - For each agent i, the number of non-zero spillover effects is bounded by a slowly growing sequence.
   - The numbers of factors are fixed.
4. Dimensional Conditions:
   - Ignoring the logarithmic factor, $\frac{\sqrt{N}}{T} \to 0$ and $\frac{\sqrt{T}}{N} \to 0$.

Theory: Assumptions

Other regularity conditions:

5. **Concentration of Random Variables**:
   - $\{G_t, \epsilon_t, U_{X,t}, U_{W,t}\}$ are sub-Gaussian.
6. **Weak Serial Dependence**:
   - $\{G_t, \epsilon_t, U_{X,t}, U_{W,t}\}$ are stationary and α-mixing with exponentially decaying mixing coefficients.
   - ϵ_t is m.d.s.
7. **Strong Factors**:
   - $\mathbb{E}[G_t G_t']$ is positive definite with bounded eigenvalues;
   - $\frac{1}{N}\lambda_i \lambda_i' \to \Sigma_\lambda$, where Σ_λ is positive definite with bounded eigenvalues; Same assumptions for loadings on X and W.

◂ Back

Outcome-specific Factors

The proposed algorithm is robust to the existence of outcome-specific factors:

$$y_{it} = x_{it}\rho + \sum_{j \neq i} \gamma_{ij} x_{jt} + W_{it}'\beta + f_t'\lambda_i + \boldsymbol{f_{y,t}}'\boldsymbol{\lambda_{y,i}} + \epsilon_{it},$$

where $f_{y,t}$ is the outcome-specific factors that do not enter the regressors X and W but are allowed to be correlated with G_t.

Outcome-specific Factors

The proposed algorithm is robust to the existence of outcome-specific factors:

$$y_{it} = x_{it}\rho + \sum_{j \neq i} \gamma_{ij} x_{jt} + W_{it}'\beta + f_t'\lambda_i + \boldsymbol{f_{y,t}}'\boldsymbol{\lambda_{y,i}} + \epsilon_{it},$$

where $f_{y,t}$ is the outcome-specific factors that do not enter the regressors X and W but are allowed to be correlated with G_t.

Extending assumptions to include outcome-specific factors,

$$\sqrt{T}\left(\hat{\gamma}_{i:}^{(db)} - \gamma_{i:}\right) = \frac{1}{\sqrt{T}}\Theta_{-i}U_{X,-i}'M_{\hat{G}}\left(F_Y\lambda_{Y,i} + \epsilon_i\right) + \mathbf{o}_p(1),$$

$$\sqrt{NT}\left(\hat{\theta} - \theta\right) = \left(\frac{1}{NT}\sum_{i=1}^{N} U_i'M_{\hat{G}}U_i\right)^{-1} \frac{1}{\sqrt{NT}}\sum_{i=1}^{N} U_i'M_{\hat{G}}\left(F_Y\Lambda_Y'M_{B,i} + \epsilon_i\right) + \mathbf{o}_p(1),$$

- The convergence rates of the original estimators does not change.
- Variances are inflated by the outcome-specific factors.

Double-refinement Algorithm

Each pass: estimate outcome-only factors from coefficient residuals, then reproject.

$$\hat{r}_i^{(s-1)} = M_{\hat{G}}\left(Y_i - X_{-i}\hat{\gamma}_{i:}^{(s-1)} - \mathbb{Z}_i\hat{\theta}^{(s-1)}\right), \qquad \hat{F}_Y^{(s)} = \mathrm{PCA}_{r_Y}\left(\hat{r}^{(s-1)}\right), \quad \hat{Q}^{(s)} = [\hat{G}, \hat{F}_Y^{(s)}].$$

0. Prelim fit

$\hat{G} \leftarrow \mathrm{PCA}(X, W)$

Plain LASSO + pooled OLS under $M_{\hat{G}}$

$\Downarrow$

$(\hat{\Gamma}^{(0)}, \hat{\theta}^{(0)})$

$\Longrightarrow$

1. Refinement 1

$\hat{F}_Y^{(1)} \leftarrow \mathrm{PCA}(\hat{r}^{(0)})$

Reproject with $M_{\hat{Q}^{(1)}}$

Adaptive LASSO selects $\hat{S}_i^{(1)}$; OLS refit gives

$\Downarrow$

$(\hat{\Gamma}^{(1,PS)}, \hat{\theta}^{(1)})$

$\Longrightarrow$

2. Refinement 2

Recompute $\hat{r}^{(1)}$ using the refined coefficients

$\hat{F}_Y^{(2)} \leftarrow \mathrm{PCA}(\hat{r}^{(1)})$

$\Downarrow$

$M_{\hat{Q}^{(2)}}$

$\Longrightarrow$

3. Inference

Debiased LASSO for spillover inference

OLS refit on $\hat{S}_i^{(1)}$ for private effects

$\Downarrow$

$(\hat{\Gamma}^{(2,db)}, \hat{\theta}^{(2)})$

Double-refinement Algorithm

Each pass: estimate outcome-only factors from coefficient residuals, then reproject.

$$\hat{r}_i^{(s-1)} = M_{\hat{G}}\left(Y_i - X_{-i}\hat{\gamma}_{i:}^{(s-1)} - \mathbb{Z}_i\hat{\theta}^{(s-1)}\right), \qquad \hat{F}_Y^{(s)} = \mathrm{PCA}_{r_Y}\left(\hat{r}^{(s-1)}\right), \quad \hat{Q}^{(s)} = [\hat{G}, \hat{F}_Y^{(s)}].$$

0. Prelim fit
$\hat{G} \leftarrow \mathrm{PCA}(X, W)$
Plain LASSO + pooled OLS under $M_{\hat{G}}$
$\Downarrow$
$(\hat{\Gamma}^{(0)}, \hat{\theta}^{(0)})$

$\Longrightarrow$

1. Refinement 1
$\hat{F}_Y^{(1)} \leftarrow \mathrm{PCA}(\hat{r}^{(0)})$
Reproject with $M_{\hat{Q}^{(1)}}$
Adaptive LASSO selects $\hat{S}_i^{(1)}$; OLS refit gives
$\Downarrow$
$(\hat{\Gamma}^{(1,PS)}, \hat{\theta}^{(1)})$

$\Longrightarrow$

2. Refinement 2
Recompute $\hat{r}^{(1)}$ using the refined coefficients
$\hat{F}_Y^{(2)} \leftarrow \mathrm{PCA}(\hat{r}^{(1)})$
$\Downarrow$
$M_{\hat{Q}^{(2)}}$

$\Longrightarrow$

3. Inference
Debiased LASSO for spillover inference
OLS refit on $\hat{S}_i^{(1)}$ for private effects
$\Downarrow$
$(\hat{\Gamma}^{(2,db)}, \hat{\theta}^{(2)})$

Why twice? Pass 1 depends on noisy preliminary coefficients; Pass 2 uses cleaner residuals, producing a better F_Y projection and lower variance.

Double-refinement Algorithm

Under additional assumptions:

1. define $d_N = \max_{l \leq N} \sum_{j=1}^{N} \mathbf{1}\{\gamma_{jl}\}$ the maximum number of receivers affected by one sender. Assume $d_N = o(N+T)$, ignoring log;
2. $\frac{T}{N} \to 0$,

we have

$$\sqrt{T}\left(\hat{\gamma}_{i:}^{(2,db)} - \gamma_{i:}\right) = \frac{1}{\sqrt{T}} \Theta_{-i} U'_{X,-i} \epsilon_i + \mathbf{o}_p(1)$$

$$\sqrt{NT}\left(\widehat{\theta}^{(2)} - \theta\right) = \left(\frac{1}{N}\sum_{i=1}^{N} \frac{1}{T} U_i' U_i\right) \frac{1}{\sqrt{NT}} \sum_{i=1}^{N} U_i' \epsilon_i + \mathbf{o}_p(1),$$

so the asymptotic results follow from the central limit theorem / Gaussian approximation.

Double-refinement Algorithm: Simulation Results

		$N = 50,\ T = 100$			$N = 50,\ T = 200$			$N = 100,\ T = 200$		
Parameter	Estimator	Bias	SD	Cov.	Bias	SD	Cov.	Bias	SD	Cov.
$\gamma = 1$	Original	−0.0064	0.0205	0.911	−0.0040	0.0136	0.940	−0.0019	0.0091	0.953
	Single refinement	−0.0075	0.0145	0.876	−0.0051	0.0098	0.882	−0.0026	0.0062	0.930
	Double refinement	−0.0037	0.0142	0.913	−0.0022	0.0096	0.925	−0.0010	0.0062	0.936
$\beta = 1$	Original	−0.0020	0.0197	0.944	−0.0017	0.0128	0.962	−0.0009	0.0091	0.962
	Single refinement	−0.0012	0.0144	0.912	−0.0013	0.0093	0.949	−0.0007	0.0065	0.943
	Double refinement	−0.0015	0.0142	0.920	−0.0016	0.0092	0.944	−0.0009	0.0065	0.946

Table 5: Private-effect estimation with an outcome-specific factor. Bias and SD are the Monte Carlo bias and standard deviation, and Cov. is the coverage rate of the nominal 95% confidence interval. Each design uses 1,000 replications.

◂ Alg. ◂ Thm.

Simulation DGP: Model and Network

One control ($K = 1$), with $(r, r_x, r_w) = (3, 1, 1)$:

$$
\begin{aligned}
Y &= X\rho + X\Gamma' + W\beta + \mathbf{1}_T a' + d\mathbf{1}_N' + f\lambda' + \epsilon, \\
X &= \mathbf{1}_T a_x' + d\eta_{d,x}' + f\eta_{f,x}' + f_x b_x' + U_x, \\
W &= \mathbf{1}_T a_w' + d\eta_{d,w}' + f\eta_{f,w}' + f_w b_w' + U_w.
\end{aligned}
$$

- $\rho = \beta = 1$;
- The network formation:

$$
\gamma_{ij} = \begin{cases} 0.5, & i \geq 2,\ |i-j| = 1, \\ 0, & \text{otherwise}, \end{cases} \qquad i, j \in \{1, \ldots, N\}.
$$

- Unit 1 receives no spillovers. Thus $\gamma_{12} = 0$ and $\gamma_{21} = 0.5$.

Simulation DGP: Factors and Loadings

Common factors. The outcome factor space is $F = [\mathbf{1}_T, d, f]$.

$$G = [d, f, f_x, f_w] = \widetilde{G}\Sigma_f^{1/2}, \qquad \widetilde{G}_{t\ell} \overset{\text{i.i.d.}}{\sim} N(1,1), \qquad (\Sigma_f)_{k\ell} = 0.5^{|k-\ell|}.$$

Here $G \in \mathbb{R}^{T\times 4}$. Factors are correlated across components and independent over time.

Outcome loadings. With $(\Sigma_N)_{ij} = 0.5^{|i-j|}$,

$$a = \Sigma_N^{1/2}\widetilde{a}, \qquad \lambda = \Sigma_N^{1/2}\widetilde{\lambda}, \qquad \widetilde{a}_i, \widetilde{\lambda}_i \overset{\text{i.i.d.}}{\sim} \mathrm{Unif}[-1,1].$$

Shared loadings induce endogeneity. Let $\Lambda = [a, \mathbf{1}_N, \lambda]$.

$$\Lambda_x = [a_x, \eta_{d,x}, \eta_{f,x}] = \Lambda + \widetilde{\Lambda}_x,$$
$$\Lambda_w = [a_w, \eta_{d,w}, \eta_{f,w}] = \Lambda + \widetilde{\Lambda}_w,$$

where the perturbation entries are i.i.d. $\mathrm{Unif}[-1,1]$.

Simulation DGP: Errors

Heteroskedastic errors and correlated regressor innovations.

$$\sigma_i^2 \overset{\text{i.i.d.}}{\sim} \text{Unif}[0.5, 2], \qquad \epsilon_{it} \sim N(0, \sigma_i^2).$$

$$\begin{pmatrix} u_{x,it} \\ u_{w,it} \end{pmatrix} = \sigma_i \widetilde{U}_{it}, \qquad \widetilde{U}_{it} \sim N\left(0, \begin{pmatrix} 1 & 0.5 \\ 0.5 & 1 \end{pmatrix}\right).$$

Errors are independent across units.

The errors in X and W share the unit-specific variance σ_i^2 and have within-unit correlation 0.5.

Simulation Design: Estimation and Inference

- 2,000 repetitions for each $(N, T) \in \{50, 200\} \times \{100, 200, 300\}$.
- PCA-Each for factors. BIC tuning for LASSO and adaptive LASSO.
- No demeaning. Factor projection removes fixed effects.
- 95% confidence intervals for $\gamma_{12} = 0$, $\gamma_{21} = 0.5$, $\gamma = 1$, and $\beta = 1$.
- Row-wise tests at 5% for $H_0 : \|\gamma_{i:}\|_\infty = 0$:
  - Unit 1: true null, so rejection frequency measures size.
  - Unit 2: $\|\gamma_{2:}\|_\infty = 0.5$, so rejection frequency measures power.
- Compare private-effect coverage with and without post-selection.

◂ Back to simulation

Simulation: RMSE of the Spillover Matrix

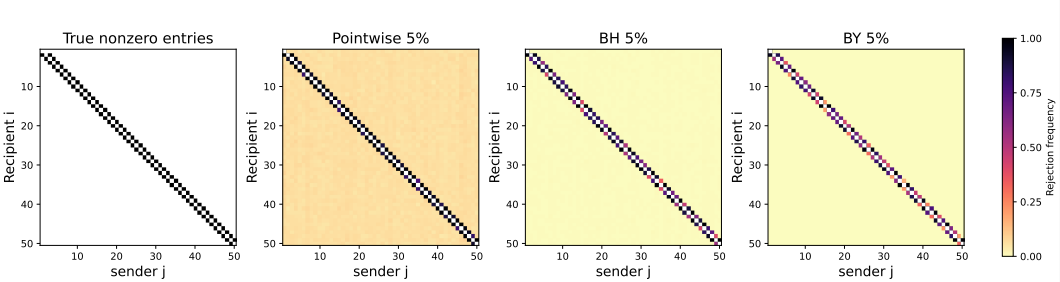
Whole-matrix RMSE over $B = 2{,}000$ repetitions:

$$\mathrm{RMSE}(\widehat{\Gamma}) = \left\{ \frac{1}{BN^2} \sum_{b=1}^{B} \|\widehat{\Gamma}^{(b)} - \Gamma\|_F^2 \right\}^{1/2}.$$

(N, T)	(50,100)	(50,200)	(50,300)	(200,100)	(200,200)	(200,300)
LASSO	0.064	0.045	0.037	0.043	0.030	0.023
Post-selection	0.058	0.031	0.020	0.041	0.022	0.013
Debiased LASSO	0.123	0.083	0.067	0.129	0.083	0.066

Back

Simulation: Network Rejection Frequencies ($N = 50$, $T = 100$)

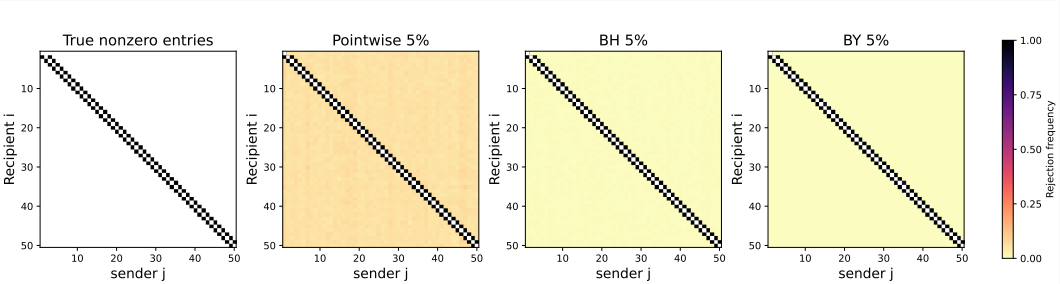


	Pointwise	BH	BY
FDR (%)	61.7	10.6	2.3
Power (%)	95.7	80.1	67.2

2,000 repetitions. Pointwise level: 5%. BH/BY FDR target: 5%. All off-diagonal pairs tested. FDR: mean false discovery proportion (zero if no rejections). Power: mean share of true links detected.

Back

Simulation: Network Rejection Frequencies ($N = 50$, $T = 300$)

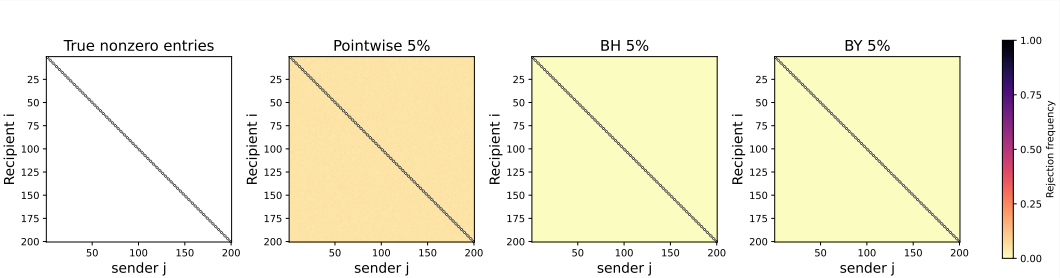


	Pointwise	BH	BY
FDR (%)	58.4	7.4	1.1
Power (%)	100.0	99.8	99.2

2,000 repetitions. Pointwise level: 5%. BH/BY FDR target: 5%. All off-diagonal pairs tested. FDR: mean false discovery proportion (zero if no rejections). Power: mean share of true links detected.

Back

Simulation: Network Rejection Frequencies ($N = 200$, $T = 300$)



	Pointwise	BH	BY
FDR (%)	84.1	6.1	0.6
Power (%)	100.0	99.1	97.6

2,000 repetitions. Pointwise level: 5%. BH/BY FDR target: 5%. All off-diagonal pairs tested. FDR: mean false discovery proportion (zero if no rejections). Power: mean share of true links detected.

‹ Back